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Packet Radio Note: R.F. Channel Capacity Considerations

Part I: Gaussian Channel Results

Shannon's capacity theorem for the Gaussian channel is not directly applicable to typical packet radio channels; however, it can provide some useful insight into some of the myriad tradeoffs which must be considered in design of a packet radio net. Furthermore, the results derived using Shannon's theorem can be adjusted to apply to practical channels.

The Capacity Theorem

Shannon's theorem states that the capacity C in bits/sec which can be achieved by a transmitter/receiver pair operating at a received signal-to-noise ratio P_R/N_oW over a bandwidth W Hz is given by

$$C = W \log_2 \left(1 + \frac{P_R}{N_o W}\right)$$

In this case P_R can be interpreted as the received signal power (average) if N_o is the (white) noise spectral power density of Gaussian noise.

Shannon has shown that this capacity cannot be exceeded for signaling at arbitrarily small message error rate, and that a signaling scheme exists which will achieve this rate at arbitrarily small error rate.

Bandwidth Limited Channels

Figure 1 is a plot of C vs. $\left(P_R/N_o\right)$ for fixed values of W. For fixed W, large P_R/N_oW C varies as the $\log_2 \left(P_R/N_o\right)$. In this region increasing the received power has very little effect on capacity, and the channel

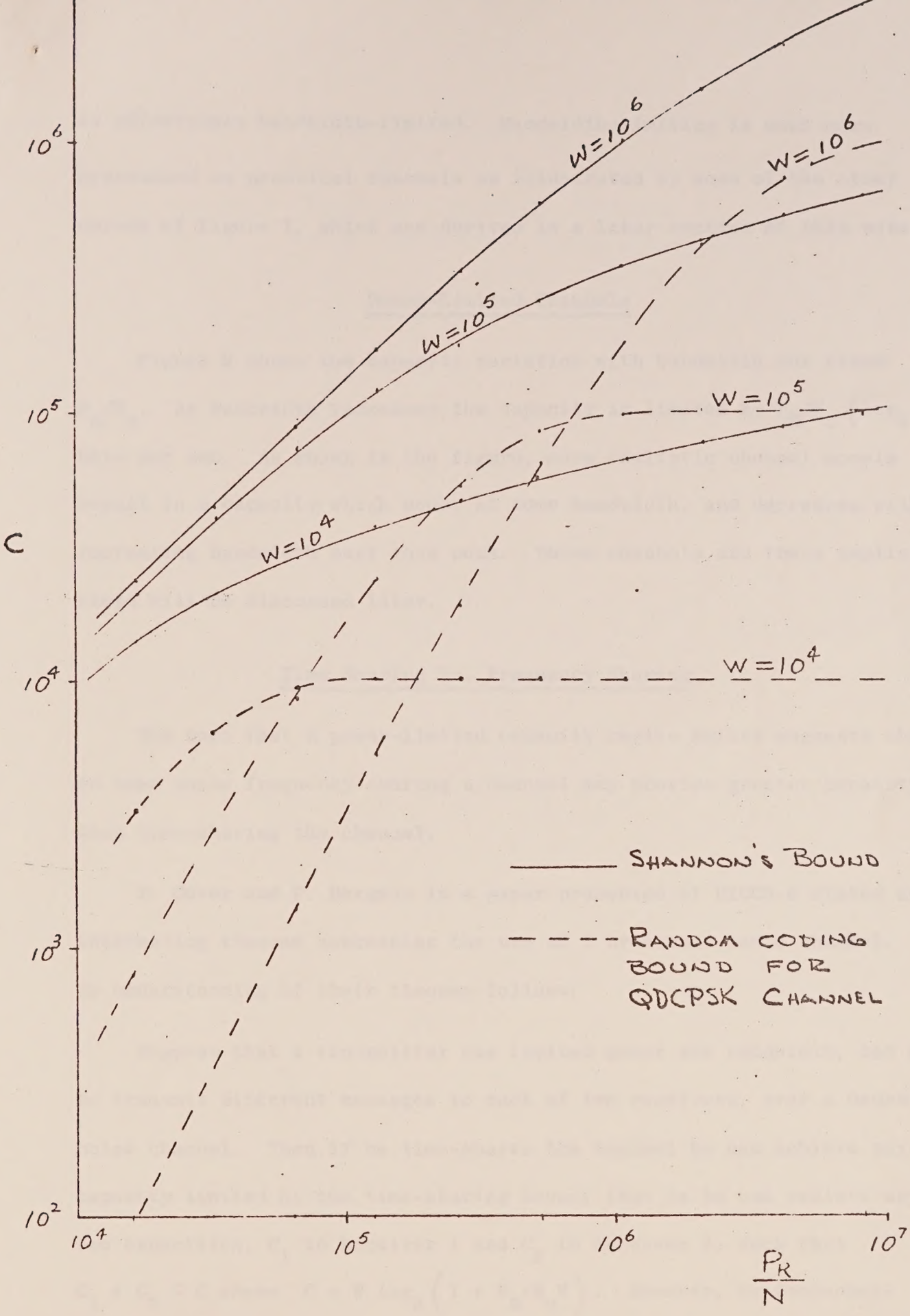

FIGURE 1 BANDWIDTH LIMITED CHANNEL CAPACITY

is effectively bandwidth-limited. Bandwidth-limiting is much more pronounced on practical channels as illustrated by some of the other curves of Figure 1, which are derived in a later section of this note.

Power-Limited Channels

Figure 2 shows the capacity variation with bandwidth for fixed P_R/N_o. As bandwidth increases the capacity is limited by $P_R/N_o \left(\log_2 e\right)$ bits per sec. As shown in the figure, more realistic channel models result in a capacity which peaks at some bandwidth, and decreases with increasing bandwidth past this peak. These channels and their implications will be discussed later.

Time Sharing Vs. Frequency Sharing

The fact that a power-limited capacity region exists suggests that in some cases frequency-sharing a channel may provide greater capacity than time-sharing the channel.

T. Cover and P. Bergman in a paper presented at HICCS-6 stated an interesting theorem concerning the use of a broadcast radio channel. My understanding of their theorem follows:

Suppose that a transmitter has limited power and bandwidth, and wishes to transmit different messages to each of two receivers, over a Gaussian noise channel. Then if he time-shares the channel he can achieve any capacity limited by the time-sharing bound; that is he can achieve any two capacities, C_1 to Receiver 1 and C_2 to Receiver 2, such that $C_1 + C_2 < C$ where $C = W \log_2 \left(1 + P_R/N_o W\right)$. However, by frequency-sharing the channel he can always achieve two capacities $\hat{C}_1$ and $\hat{C}_2$

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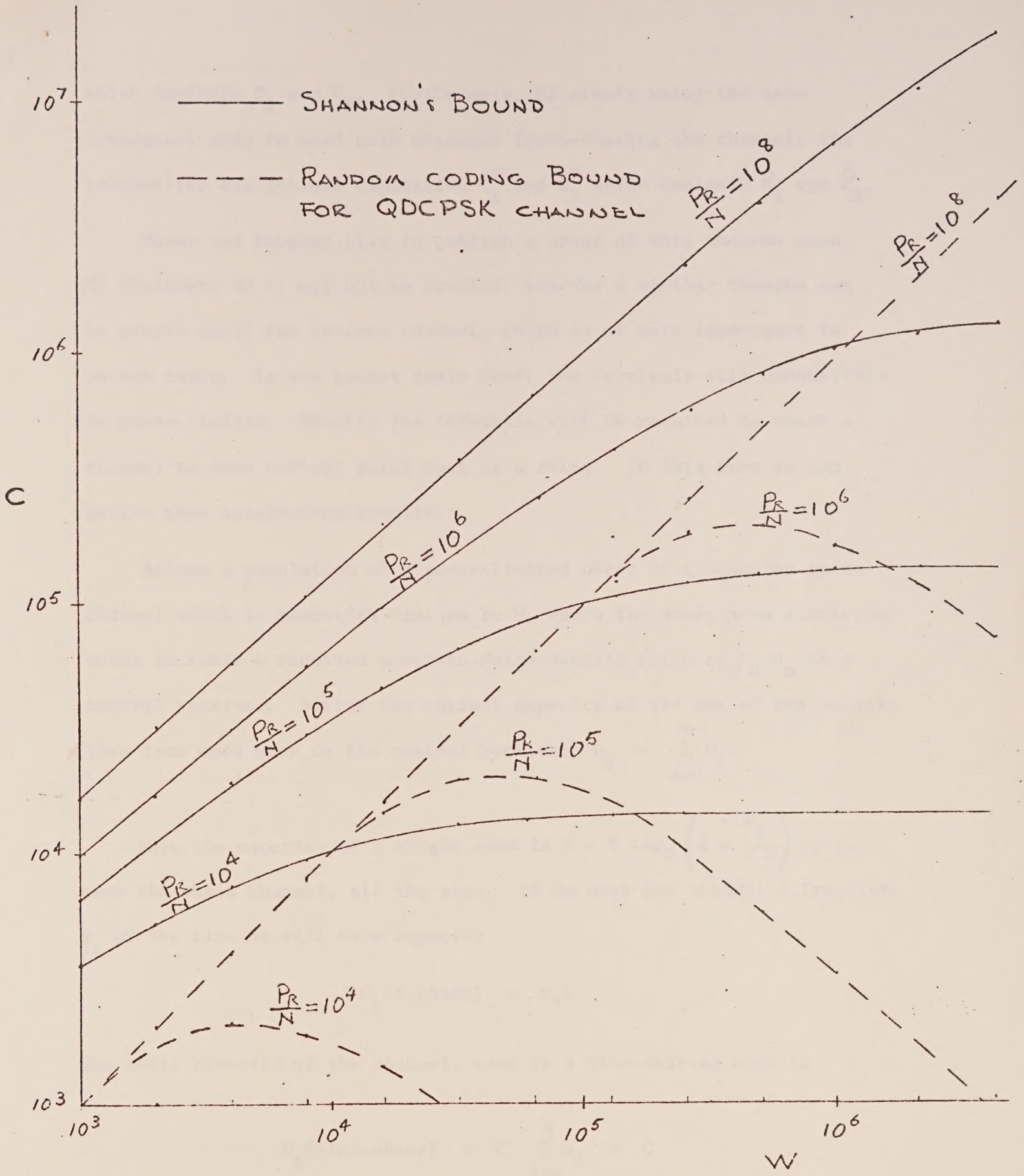

FIGURE 2 POWER LIMITED CHANNEL CAPACITY

which dominate C_1 and C_2. Furthermore, by simply using the same orthogonal code to send both messages (code-sharing the channel) the transmitter can achieve capacities C_1^* and C_2^* which dominate $\hat{C}_1$ and $\hat{C}_2$.

Cover and Bergman plan to publish a proof of this theorem soon. My statement of it may not be precise; however a similar theorem can be proven about the inverse channel, which is of more importance to packet radio. In the packet radio case, the terminals will undoubtedly be power-limited. Usually the terminals will be required to share a channel to some central point such as a relay. In this case we can derive some interesting results.

Assume a population of n power-limited users of a Gaussian noise channel which is bandwidth limited to W, where the users have sufficient power to cause a received power-to-noise-density ratio of P_R/N_o at a central receiver. Define the channel capacity as the sum of the capacities from each user to the central receiver, $C_S = \sum_{i=1}^{n} C_i$.

Then the capacity of a single user is $C = W \log_2 \left(1 + \frac{P_R}{N_o W}\right)$ if he uses the whole channel, all the time. If he uses the channel a fraction α_i of the time he will have capacity

$$C_i(\text{t-share}) = \alpha_i C$$

the total capacity of the channel, used in a time-sharing mode is

$$C_S(\text{time-share}) = C \sum_{i=i}^{n} \alpha_i \leq C$$

On the other hand, if each user uses α_i of the frequency band, he will have capacity

$$C_i(\text{f-share}) = \alpha_i W \log_2 \left(1 + \frac{P_R}{N_o W \alpha_i}\right)$$

Note that

$$\frac{C_i(\text{f-share})}{C_i(\text{t-share})} = \frac{\alpha_i W \log_2 \left(1 + \frac{P_R}{N_o W \alpha_i}\right)}{\alpha_i W \log_2 \left(1 + \frac{P_R}{N_o W}\right)}$$

$$= \frac{\log_2 \left(1 + \frac{k}{\alpha_1}\right)}{\log_2 (1 + k)}$$

Since $0 < \alpha_i \leq 1$

$$C_i(\text{f-share}) \geq C_i(\text{t-share})$$

Hence $C_S(\text{f-share}) \geq C_S(\text{time-share})$, so that frequency-sharing theoretically dominates time-sharing for the inverse broadcast channel. This comes about because more power is available, on the average, in the frequency-sharing mode since we have limited the peak-power of the individual transmitters. Of course, if code-sharing is used the channels will be orthogonal, and the total capacity $C_S(\text{code-share}) = nC > C_S(\text{f-share})$ can be achieved.

Practically, code-sharing requires that all users be phase-synchronous and that the central receiver must consist of n receivers; frequency-sharing requires guard-bands so that some efficiency is lost; and time-sharing requires time-guard bands so that some efficiency is lost.

It is possible to combine frequency and time-sharing in order to ease the implementation problem. If we divide the channel into m sub-channels and time-share the subchannels, then a capacity of

$$\hat{C}_S = \sum_{i=1}^{n/m} \alpha_i \frac{W}{m} \log_2 \left(1 + \frac{P_R m}{N_o W}\right) \leq W \log_2 \left(1 + \frac{P_R m}{N_o W}\right)$$

if the channel is power-limited, so that

$$\frac{P_R m}{N_o W} < 1$$

then

$$\hat{C}_S \cong m \frac{P_R}{N_o} \log_2(e)$$

This is an increase by a factor of m over simple time-sharing.

If the channel is bandwidth-limited, so that $P_R/N_o W > 1$, then frequency-sharing will not be significantly better than time-sharing.

This suggests that if the terminal power, range, noise, etc. are fixed, and we have the opportunity to use more bandwidth, we should time-share the band until we are power-limited; i.e., reach a signal-to-noise ratio of about 1 $\left(P_R/N_o W = 1\right)$. At that point we should start splitting the band into sub-channels; i.e., we always maintain the signal-to-noise ratio greater than 1. All of the example calculations by T. Corey in PRN #4 result in $P_R/N_o W = 20$. In these cases, if we could achieve Shannon's bound, we could operate just as efficiently with time-sharing as by frequency-sharing.

Practical channels reach the power-limited region at much higher signal-to-noise ratios (smaller bandwidths) than the Gaussian channel, so that band-splitting will be efficient at smaller bandwidths.

L.O.S. Range

Shannon's bound can also provide some insight into the question of L.O.S. range. Suppose that we assume that a single repeater must serve an area in which there are many terminals. Let

δ = density of terminals

S = source rate per terminal (bits/sec)

U = channel utilization

Then the capacity needed in a single repeater

$$C_N = \delta \pi R^2 \frac{S}{U}$$

to handle all terminals in an area of radius R.

The typical signal strength available from a terminal at range R varies as $\frac{1}{R^4}$. Thus the capacity available within a circle of radius R is

$$C_A = W \log_2 \left(1 + \frac{P}{N_o W R^4}\right)$$

where P is the power received at 1 mile.

It is logical to choose an L.O.S. range so that $C_A = C_N$ (ignoring interference). This can be found by plotting C_A and C_N vs. R, as in Figure 3.

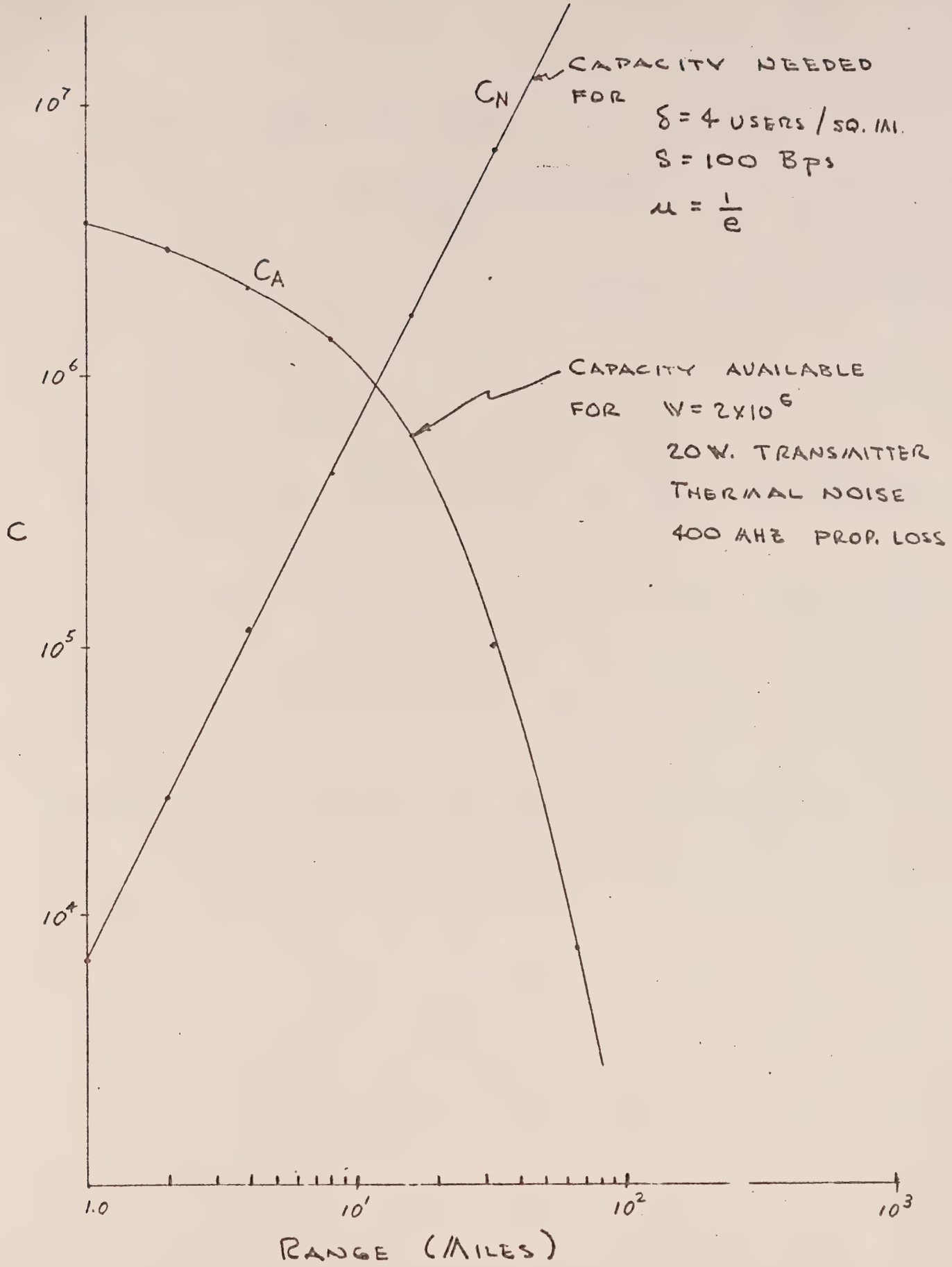

FIGURE 3 CAPACITY VS. MAX. RELAY-TERMINAL RANGE

In this figure we have assumed

$$\delta = 4 \text{ terminals/mile}$$

$$S = 100 \text{ bps}$$

$$U = 1/2\ e$$

$$W = 2 \times 10^5 \text{ Hz}$$

$$P/N_o = 9 \times 10^{10}/R_4$$

This corresponds to a terminal transmitter power of about 20 watts at 500 MHz depending on antenna design, etc. (PRN #4 example uses 4 W at 500 MHz to achieve this P/N_o).

The results from this figure would indicate that a range of about 10 miles is suitable for this example. We should not take this number too seriously until a more realistic channel model is introduced; however the computation illustrates clearly that terminal peak power, terminal density, source bit rate, bandwidth and utilization are all interrelated in a rather simple and direct way in determining L.O.S. range.

Part II: More Realistic Channels

The capacity of less ideal channels is not so easily calculated. Several realities must be considered:

1. The real channel will include additive non-Gaussian as well as Gaussian noise.
2. The real channel will have random amplitude (space-fading).
3. The real channel will have multipath effects.
4. Modulation and coding schemes to achieve capacity are too complex and expensive for implementation.

Although the first effect will be very important, we must defer consideration of non-Gaussian noise to another note, and restrict this note to Items 2, 3, and 4 above.

An excellent discussion of these considerations is given in Wozencraft and Jacobs,[1] and need not be repeated here. More to the point is a specialization to the packet radio problem. For this discussion we introduce the functional block diagram of the channel shown in Figure 4.

The transmitter must accept a message from the source and select a waveform to represent the message. For our purposes a message is a packet. The waveform-selection process can be broken into two steps: encoding and modulation.

The waveform is frequency-translated, amplified and radiated. The propagation channel distorts the waveform and adds noise. The receiver uses the distorted, noisy replica to try to decide which message was

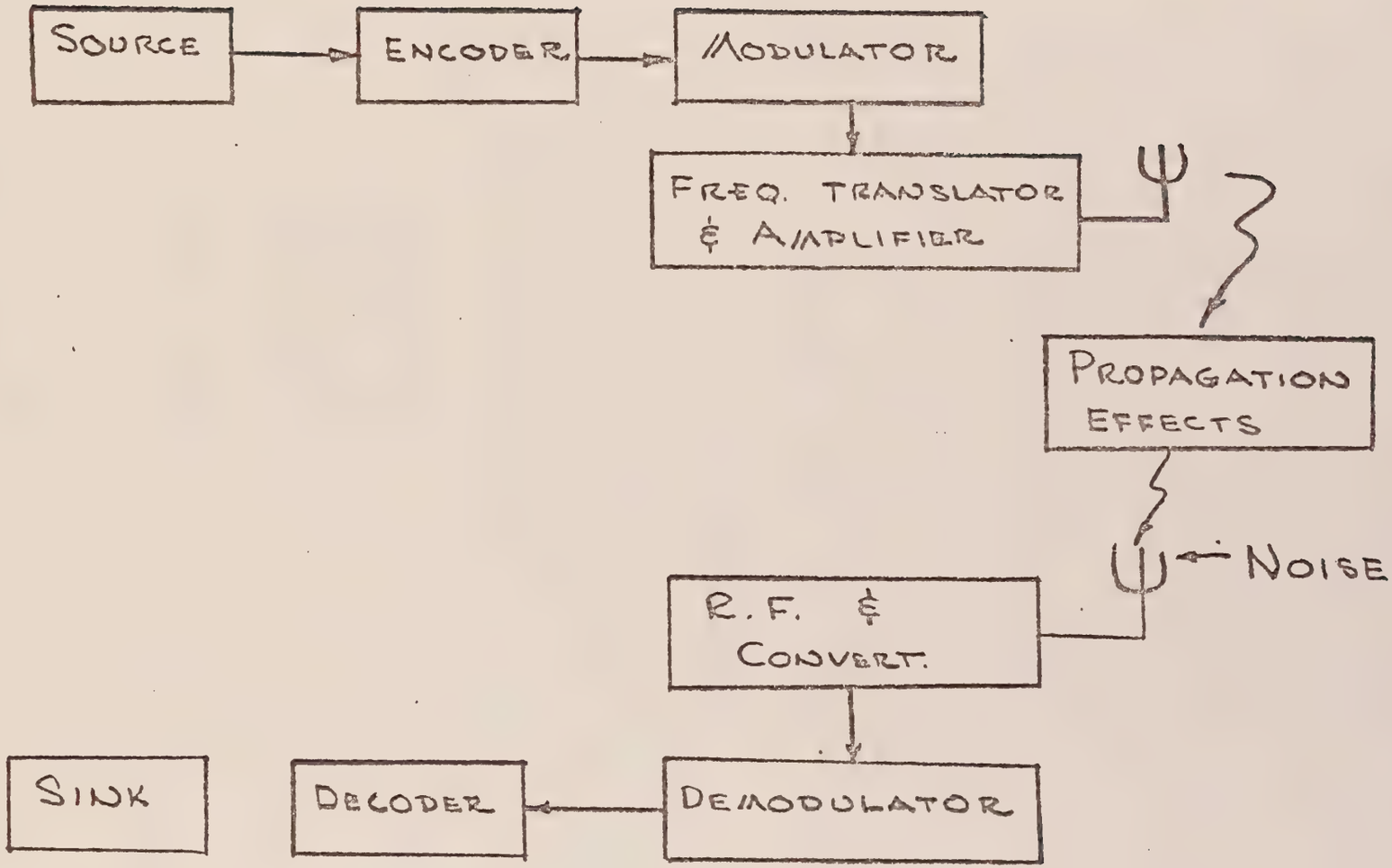

FIG 4- FUNCTIONAL BLOCK DIAGRAM OF CHANNEL

sent. In the most general case it is not possible to break the decision process into demodulation and decoding, but for our purposes this breakdown will be justified.

Design of the rf part of the system will require selection of the coder/decoder and modulator/demodulator as well as selection of frequency, peak power, antennas, etc. Selection of coder and modem should be made to best use the propagation channel.

Our purpose here is to discuss some of the important considerations for rf design and estimate their relation to the maximum rate at which messages can be communicated with some specified error rate.

A Low-Risk Channel

Before discussing more realistic achievable rates, it is worthwhile to calculate the rate which can be achieved with a simple low-technical risk terminal-to-relay link. For this purpose we define such a link below:

Transmitter Power	13 dBW (20 W)
Transmitter Antenna Gain	2 dB
Receiver Antenna Gain	4 dB
Receiver Noise Figure	7.8 dB
Operating Frequency	400 MHz
Modulation Type:	Noncoherent F.S.K.

Propagation Considerations

In order to obtain a conservative bound on link performance, we assume that the link operates in an urban area where the propagation losses vary with range as $1/R^4$ and the received field strength at

range R is a random variable with a Rayleigh distribution. The mean propagation loss is taken from packet radio Note 4 as 145.6 dB at 10 miles.

Measurements made by Turin[2] and Schmid[3] indicate that the multipath delay-spread in an urban/suburban area is 4 to 10 μs (let us assume 5 μs for this note). So long as the bit durations exceed about twice the delay-spread, performance will not be significantly degraded by multipath; hence FSK bits rates up to 100 kbps can be supported without resorting to any form of multipath-equalization. (Higher bit rates are possible with other techniques - see below.)

Communication Rate

Suppose that we define acceptable performance in terms of the probability that a 1,000 bit packet will be incorrectly received. If we require a packet error rate less than 1 in 10^{-2}, and if we assume independent bit errors, then we will require a bit error rate of less than 1 in 10^{-5}. A non-coherent FSK modem will provide this error rate for all SN Ratios exceeding about 13 dB if the noise is Gaussian and there is no fading or multipath distortion. Although the received signal strength in an urban area varies randomly as the receiving antenna is moved over a small area, it does not fluctuate in time, except very slowly, so long as the receiving antenna is stationary. We assume that the slowly-moving user moves his antenna to avoid deep nulls, or that he is provided with some form of diversity to overcome the deep nulls. That is, we ignore the fading effects. In this case, a 10^{-2} message error rate will be achieved so long as

$$\frac{E_S}{N_O} \geq 23$$

For the 20 W transmitter assumed,

$$\frac{P_R}{N_O} = \frac{9 \times 10^{10}}{R^4}$$

For rectangular pulses of duration T, and amplitude A

$$P_R = \frac{A^2}{2} \quad \text{and} \quad E_S = \frac{A^2 T}{2} = P_R T \ .$$

Hence $$\frac{P_R T}{N_O} \geq 23 \quad \text{or} \quad T \geq \frac{23}{9 \times 10^{10}}$$

so that the maximum bit rate will be

$$B = \frac{1}{T} \leq \frac{4.10^9}{R^4} \ .$$

Considering multipath to provide an upper limit of 100 kbps, the achievable bit rate for a low-risk link is:

$$B \leq \min \left(10^5, \frac{4 \times 10^9}{R^4} \right)$$

Figure 5 shows B versus R for the non-coherent FSK link described above. The bit rate is multipath limited out to a range of about 14 miles.

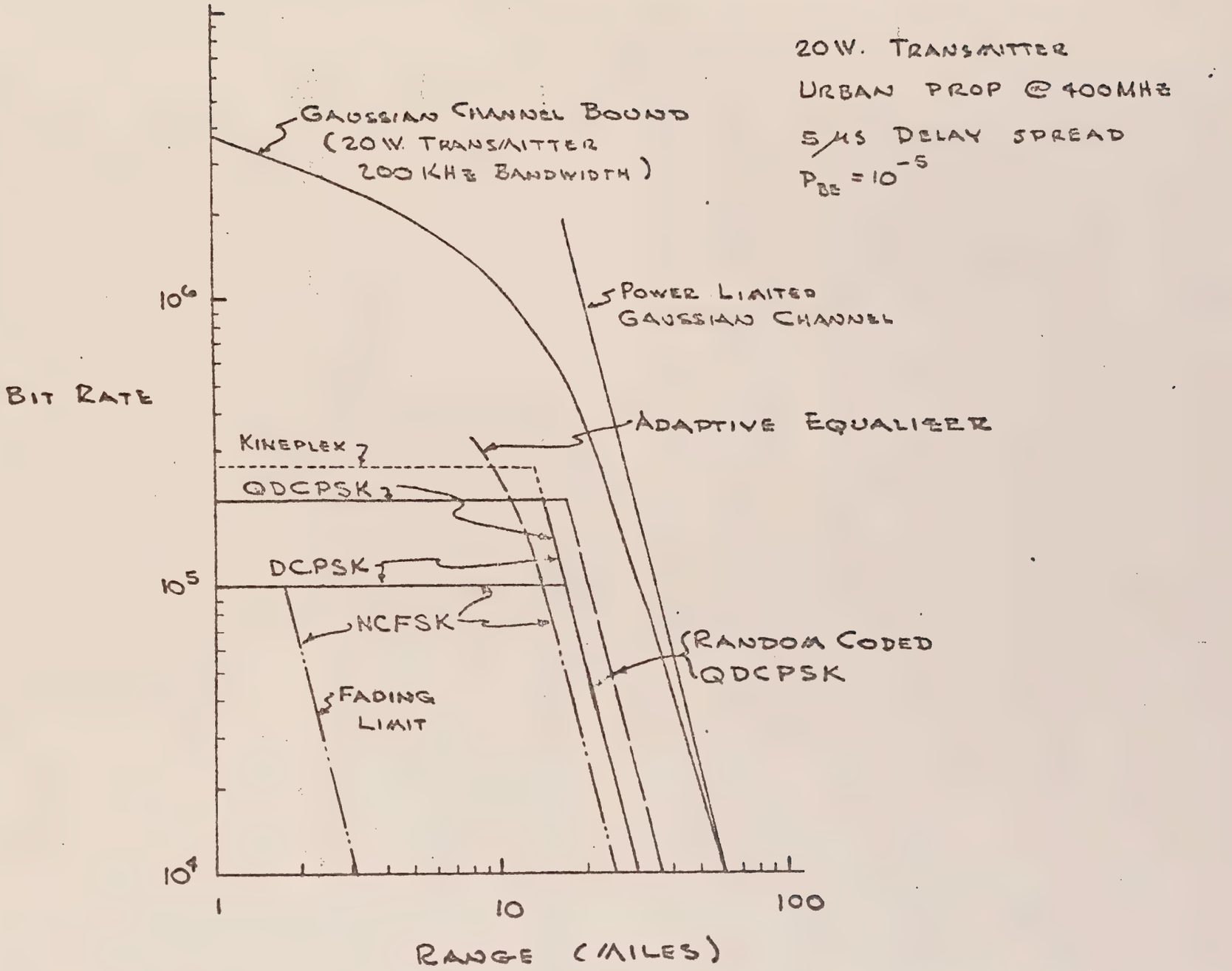

FIG 5 - SUPERCURVES (MAX. BIT RATE VS. RANGE).

Beyond this range it becomes power-limited, and bit rate decreases rapidly with range.

An FSK signal occupies about 3B Hz where B is the bit-rate. Thus in the multipath-limited region the width of the frequency band used by this low-risk link is about 300 kHz. For comparison we have shown Shannon's bound for a 200-kHz bandwidth Gaussian channel, where the same transmitter power, propagation losses, etc., have been assumed. It is clear that the non-coherent FSK system can be improved upon.

To show that things could be worse, the curve for FSK performance over a fading channel is also shown. This would apply if, for example, the users moved rapidly, or were not allowed to avoid nulls.

Improved Channels

Also shown on Figure 5 are curves of achievable bit rate versus range for a variety of improvements. Techniques to use power more efficiently include better modulation, such as differentially coherent phase shift keying (DCPSK) with two or four phases (QDCPSK) and the application of error correction coding. Both DCPSK and QDCPSK have error probabilities given by

$$P_e = 1/2 \exp\left(\frac{-P_R T}{N_O}\right)$$

where T = duration of one bit

hence they require 3 dB less power than NCFSK. Coherent PSK is not shown because it is more complicated to receive than DCPSK, and it does not provide significant improvement at the required message error rate.

The random coding curve for DCPSK with a quantized receiver can be found by considering the capacity of a binary-symmetric channel with error probability p:

$$C = B \left[1 - \log_2 \left(1 + 2\sqrt{p(1-p)}\right)\right]$$

If the expression for P_e is substituted in this equation we can obtain an expression for C in terms of B and $P_R/N_O R^4$. For a fixed $P_R/N_O R^4$ there is an optimum value of B which we found by plotting a series of curves of C versus B. The curve shown on Figure 5 is the maximum value of C for each R, with P_R/N_O fixed.

Practically, it is very difficult to achieve this curve, since it requires complex decoding, and assumes infinite block lengths. On the other hand the performance of an uncoded QDCPSK system is almost as good and relatively easily achieved.

We should note that in the power-limited region there are many types of modulation which reduce to DCPSK or QDCPSK and will provide equivalent performance. Among these are Kineplex, MSK, and several varieties of spread-spectrum modulation.

At shorter ranges, each modulation scheme runs into a multipath limit. We have shown this on the curves at a delay-apread of 5 μs assuming that intersymbol interference becomes too great whenever a symbol is shorter than twice the delay-spread. This is probably a conservative bound, and will be investigated in more detail, but it is probably not off by more than a factor of 2.

The multipath limit is raised by modulation schemes which transmit multiple bits per symbol. The limit for QDCPSK is just twice that for DCPSK because the 4-phase signal carries 2 bits per symbol. Also shown is the limit for a 16-tone QDCPSK Kineplex modem. In this case one pulse on each tone carries 2 bits, and since there are 16 tones, there are 32 bits per symbol; however, for acceptable operation the delay spread cannot exceed about 2 percent of the symbol duration. Thus the multipath limit is about 260 kBps. Lest someone think that this is too favorable a comment on Kineplex (invented by Collins) I should note that it is a complex modem to construct, and I am not sure that anyone has built one at bit rates exceeding about 2×10^4 B ps. Kineplex is like MSK in that it achieves efficient use of the bandwidth, since it achieves roughly one bit per Hz. QDPSK can be filtered to approach this, but usually takes slightly more than one Hz per bit. DCPSK uses roughly 2 Hz per bit, orthogonal FSK uses about 3 Hz per bit.

Modulation schemes which perform at rates exceeding 1 bit per Hz are possible. Such schemes require multi-level symbols. These are undesirable for the urban channel where amplitude will be random; however, at very high received power levels (short range) it might be possible to use multilevel signals. It would probably be necessary to equalize the channel to use such schemes. If the user moved slowly so that the channel did not vary from bit to bit, an adaptive equalizer might be possible. The use of such an equalizer should be studied if operation at short ranges (or with higher power) is considered.

A curve showing the performance achievable with an adaptive equalizer is also shown in Figure 5. This was found from Monson's[4]

results simulating a decision-feedback equalizer in multipath. The modulation scheme assumed by Monson is binary phase-shift keying with coherent demodulation. This modulation technique cannot be used on the packet radio channel where the received phase is unknown, but for low error probabilities DCPSK performance should be approximately the same.

Conclusions

Examination of supercurve leads to some interesting conclusions about the rf design.

a. The terminal-to-relay link will be either power-limited or multipath limited depending upon the terminal power, and the ratio of the terminal density-source bit rate product to channel utilization. For practical terminals this break-point will occur when the $\delta S/U$ ratio is about 300.

b. For smaller ratios the link will be power limited and only an increase in power or an improvement in modulation scheme will improve performance. In this region the modulation technique should probably be some version of QDCPSK (including Kineplex, MSK, and Spread Spectrum formats).

c. For larger $\delta S/U$ ratios the link will be either bandwidth limited or multipath limited, depending upon the available bandwidth. In this region only the use of anti-multipath techniques will improve performance. Use of modulations transmitting multiple bits per symbol such as QDPSK or Kineplex is important in this region, since such signals can increase the bit rate by a factor of 2 or more. Adaptive equalizers might increase bit rate by an additional factor of 1.5 to 2. This amounts to an increase in the relay range by a factor of 1.2 to 1.4. Anti-multipath techniques may not be worth the complexity, and a probable upper bound on the practical bit rate is 200 kBps.

d. If we are power-limited to 20 W and have 200 kHz available, then so long as our user distribution is such that we wish to operate each relay at ranges less than about 15 miles we should use a single frequency per relay.

e. If our user density is such that longer ranges are desired, then we should use two frequencies per relay at 15 miles, three at 19 miles, etc.

f. In general, for a bandwidth W, a range R, and a power P_R we can find curves of maximum bit rate versus range such as Figure 5. We will best use the bandwidth and power if we use n channels when the (QDCPSK) maximum bit rate is between W/n and W/n+1. Of course complexity considerations may override considerations of efficient channel bandwidth usage.

g. It is of critical importance to the RF design to determine the peak power available in a hand-held terminal, the bandwidth which might be allocated to such a system, and the user geographic distribution and average bit rate.

Rash Predictions and Weasel Words

It seems likely that if we assume a bit rate of 100-200 kBps and make sure that our L.O.S. range does not exceed 10 miles, we will be able to construct a hand-held terminal to operate in the urban environment. The terminal will use some form of multiple bit per symbol modulation, and the relay will have some form of diversity reception.

All of this may be changed if the user requirements (terminal density or source bit rate) change or if the effects of man-made noise are significant at the frequency finally chosen for operation.

REFERENCES

1. J. Wozencraft and I. Jacobs, Principles of Communication Engineering, John Wiley and Sons Incorporated, New York, New York (1965).

2. G. L. Turin et al., "A Statistical Model of Urban Multipath Propagation," IEEE Trans. on Vehicular Technology, Vol. VT-21, pp 1-9 (February 1972).

3. H. F. Schmid et al., "A Review of the U.S. Army Random-Access Discrete-Address System Program and the Martin Company Design Plan," Vol. 2, Systems Techniques Laboratory, Stanford Electronics Laboratories, Stanford, California, Technical Report #2401-1, (May 1967).

4. P. Monson, "Digital Transmission Performance on Fading Dispersive Diversity Channels," IEEE Trans. on Comm. Tech., Vol. Com-21, pp 33-39 (January 1973).